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UNSTEADY RADIATIVE–CONVECTIVE HEAT TRANSFER IN A HIGH-TEMPERATURE GAS–PARTICLE FLOW PAST A SEMI-TRANSPARENT PLATE

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Numerical simulations of unsteady radiative-convective heat transfer in a turbulent flow of a mixture of gases and solid particles past a semi-transparent plate are performed. An ablation process is demonstrated to occur on the plate surface in the case of intense radiative heating of the plate by an external source with emission in a limited spectral range. Temperature fields and distributions of heat fluxes in the boundary layer and in the plate are calculated. Calculation results are presented, which allow determining the effect of ablation and reflecting properties of the plate surface on the thermal state of the medium in the system containing the boundary layer and the plate under conditions of plate heating by a high-temperature source of radiation.

Key words: *radiation, turbulence, boundary layer, ablation, scattering.*

The process of radiative–convective heat transfer on a porous plate with gas injection through the plate was studied in [1–3]. The injected flow was independent of the plate temperature and was defined *a priori*. In [4], the process of mass supply through the surface into the boundary layer was studied in combination with heat transfer; an ablating plate model was used. Heat transfer in the flow past a semi-transparent plate in the absence of particles in the boundary layer and ablation on the surface was calculated in [5].

In the present study, we consider an adjoint problem of radiative–convective heat transfer in a turbulent flow of an emitting–absorbing and scattering gas–particle medium past a semi-transparent ablating plate. For simplicity, we assume that the vapors of the plate material do not affect the optical and thermophysical properties of the medium, while the presence of particles in the flow does not affect the thermophysical properties and determines the optical properties. The particle size remains unchanged in the course of heat transfer. The optical properties of the medium depend on temperature and radiation wavelength. The specific heat is assumed to be constant, the viscosity and thermal conductivity are linear dependences of temperature, and the density is an inverse dependence of temperature. Heat transfer inside the plate through radiation and heat conduction is taken into account in the direction perpendicular to the plate surface. The optical properties of the plate material depend on the wavelength, and the thermal conductivity is temperature-dependent. The time of heating of the boundary layer is assumed to be much shorter than the time of heating of the plate; hence, heat transfer in the boundary layer can be considered in a quasi-steady approximation. The initial value of the plate temperature is T_{w0}; this value is maintained constant in the region $0 < x < x_0$ during the entire heating process. The lower surface of the plate is thermally insulated. The source of radiation, which is a blackbody with a temperature T_s, is located outside the boundary layer and emits in a limited spectral range Δ. The medium of the boundary layer is emitting, absorbing, and scattering, while the medium of the plate is emitting and absorbing. The source surface is parallel to the plate surface.

Under the assumptions used, the velocity field in the boundary layer is described by the differential equation

$$((1+\bar{\mu}_t)f'')' + \frac{1}{2}ff'' = \xi\Big(f'\,\frac{\partial f'}{\partial \xi} - f''\,\frac{\partial f}{\partial \xi}\Big) \qquad (1)$$

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with the boundary conditions

$$\eta = 0:\quad f = 0,\quad f' = -f_w,\qquad \eta \to \infty:\quad f' \to 1,$$

where f is the dimensionless stream function, $f_w = V_w(\mathrm{Re}\,\xi)^{1/2}$, $V_w = \rho_w v_w/(\rho_\infty u_\infty)$ is the dimensionless mass flow on the plate surface, the subscripts w and "∞" correspond to conditions on the plate and in the external flow, $\eta = \Big(\dfrac{\rho_\infty u_\infty}{\mu_\infty x}\Big)^{1/2} \displaystyle\int_0^y \dfrac{\rho}{\rho_\infty}\,dy$ and $\xi = x/L$ are the transverse and longitudinal dimensionless coordinates, x and y are the corresponding dimensional coordinates, u and v are the streamwise and crossflow components of velocity, respectively, ρ is the density, μ is the viscosity, L is the length of the calculated region of the plate, $\mathrm{Re} = \rho_\infty u_\infty L/\mu_\infty$ is the Reynolds number, and the prime means differentiation with respect to the coordinate η.

The thermal part of the problem consists of equations and boundary conditions that describe heat transfer in the boundary layer

$$\frac{\partial}{\partial\eta}\Big(\Big(\frac{1}{\mathrm{Pr}} + \frac{\bar\mu_t}{\mathrm{Pr}_t}\Big)\frac{\partial\theta}{\partial\eta}\Big) + \frac{f}{2}\frac{\partial\theta}{\partial\eta} - \xi f'\frac{\partial\theta}{\partial\xi} - \frac{\mathrm{Sk}}{\mathrm{Re\,Pr}}\,\xi\Psi = 0,$$

$$\xi_0 < \xi < \xi_1,\qquad 0 < \eta < \infty,$$

$$\xi = \xi_0:\qquad \theta = \theta_0, \tag{2}$$

$$\eta = 0:\quad \theta = \theta_w,\qquad \eta \to \infty:\quad \theta \to 1$$

and in the plate

$$\frac{\partial\theta_w}{\partial\,\mathrm{Fo}} = \frac{\partial}{\partial\zeta}\Big(\Lambda\frac{\partial\theta_w}{\partial\zeta}\Big) + \mathrm{Sk}_w\frac{\partial\Phi_w}{\partial\zeta},\qquad 0 < \zeta < 1,\quad \mathrm{Fo} > 0; \tag{3}$$

$$\mathrm{Fo} = 0,\ 0 \leqslant \zeta \leqslant 1:\quad \theta_w = \theta_{w0};$$

$$\zeta = 0,\ \mathrm{Fo} > 0:\quad \Lambda\frac{\partial\theta_w}{\partial\zeta} = \mathrm{Sk}_w(Q - \Phi_w); \tag{4}$$

$$\zeta = 1,\ \mathrm{Fo} > 0:\quad \frac{\partial\theta_w}{\partial\zeta} = 0.$$

[Note that Eq. (4) is the condition of matching of the heat flux on the interface between the boundary layer and the plate.] In relations (1)–(4), $\bar\mu_t = \mu_t/\mu$ (μ_t is the turbulent viscosity), $\theta = T/T_\infty$, $\theta_w = T_w/T_\infty$ (T and T_w are the temperatures in the boundary layer and in the plate, respectively), $\theta_0(\eta)$ is the self-similar solution of the energy equation (2) with radiation being ignored, $\Phi_w = E_w/(4\sigma T_\infty^4)$, E_w is the integral (over the spectrum) density of the resultant radiation flux in the plate, $\zeta = y/H$ (H is the plate thickness), $\mathrm{Fo} = a_c t/H^2$ is the Fourier number, $\mathrm{Pr} = \mu_\infty/(\rho_\infty a_\infty)$ is the Prandtl number, $\mathrm{Sk} = 4\sigma T_\infty^3 L/\lambda_\infty$ and $\mathrm{Sk}_w = 4\sigma T_\infty^3 H/\lambda_\infty$ are the Stark numbers in the boundary layer and in the plate, respectively, Pr_t is the turbulent Prandtl number, $\Lambda = \lambda_c/\lambda_\infty$ (λ_c and λ_∞ are the thermal conductivities of the plate material and of the medium in the external flow, respectively), a_c and a_∞ are the thermal diffusivities of the plate material and of the medium in the external flow, $\xi_0 = x_0/L$ and $\xi_1 = x_1/L$ (x_0 and x_1 are the boundaries of the calculated region of the plate), and σ is the Stefan–Boltzmann constant.

The dimensionless density of the total heat flux on the plate surface Q in Eq. (4) is determined by the expression

$$Q = -\frac{1}{\mathrm{Sk}}\Big(\frac{\mathrm{Re}}{\xi}\Big)^{1/2}\Big(\frac{\partial\theta}{\partial\eta}\Big)\Big|_{\eta=0} + \Phi - \frac{\mathrm{Re\,Pr}}{\mathrm{Sk}}V_w Q_L,$$

where $\Phi = E/(4\sigma T_\infty^4)$, E is the integral (over the spectrum) density of the resultant radiation flux in the boundary layer, and $Q_L = q_L/(\rho_\infty c_p T_\infty)$ (q_L is the heat of evaporation of the plate material). The expression for dimensionless divergence of radiation flux density in Eq. (2) has the form

$$\Psi = \int_\Delta \frac{\tau_{\lambda L}(E_{0\lambda} - E_{*\lambda})}{4\sigma T_\infty^4}\,d\lambda,$$

where $E_{0\lambda}(T)$ is the density of the equilibrium radiation flux, $E_{*\lambda} = 2\pi \int\limits_{-1}^{1} I_\lambda(\tau_\lambda, \chi)\chi\, d\chi$ is the volume density of the incident radiation flux, I_λ is the radiation intensity, χ is the cosine of the angle between the ordinate axis and the direction of radiation propagation, λ is the wavelength, $\tau_{\lambda L} = k_\lambda L$ is the characteristic optical thickness, and k_λ is the attenuation coefficient of the medium; the subscript λ corresponds to spectral quantities. The optical thickness in the cross section ξ of the boundary layer is a function of the wavelength and temperature; it can be presented in the form

$$\tau_\lambda = \Big(\frac{\xi}{\mathrm{Re}}\Big)^{1/2} \int\limits_0^\eta \frac{\tau_{\lambda L}}{\theta}\, d\eta.$$

The radiative system considered consists of two flat layers. The first layer is located between the surfaces of the external source of radiation and the plate. This layer contains the emitting–absorbing and scattering medium of the boundary layer. The second layer is the semi-transparent emitting–absorbing plate. The interface between the layers is assumed to be transparent and to ensure diffuse and specular reflection. The equation of radiation transfer was solved by the method of mean fluxes [6]. Effects of refraction and complete internal reflection on the inner boundary of the plate were taken into account on the interface between the boundary layer and the plate [5]. The velocity field in the turbulent boundary layer was calculated by the two-layer Cebeci–Smith model [7]. After the joint solution of Eqs. (1)–(3) and the radiation transfer equation, we found the velocity field in the boundary layer and the temperature fields in the boundary layer and in the plate. The calculation was performed by an iterative difference method.

The gas–particle medium under study was a mixture of carbon dioxide, water vapor, and solid-phase particles, which were coal particles. To a certain extent, such a mixture allows modeling the atmosphere of furnaces of steam boilers.

Neglecting scattering in the gas phase, we can present the attenuation coefficient of the considered model medium in the form

$$k_\lambda = k_{\lambda p} + \varkappa_{\lambda g},$$

where $k_{\lambda p}$ is the attenuation coefficient of the cloud of particles and $\varkappa_{\lambda g}$ is the absorption coefficient of the gas.

To take into account selective absorption of radiation in the gas phase, we used the narrow band method based on the Goody statistical model [8], which implies that the absorption lines in the frequency spectrum are distributed randomly, and the line intensity is distributed in accordance with a certain law. An exponential distribution is usually used. Within the framework of the narrow band method, the spectral absorption coefficient at moderate pressures can be presented as

$$\varkappa_{\lambda g} = P(\gamma_{\lambda \mathrm{CO_2}} C_{\mathrm{CO_2}} + \gamma_{\lambda \mathrm{H_2O}} C_{\mathrm{H_2O}}),$$

where P is the total pressure of the gas, C are the molar fractions of the species, and γ_λ is the temperature-dependent mean intensity of the line in the absorption band.

In the present work, we used the values of the parameter γ_λ obtained in [9–11] for the temperature range from 300 to 1500 K. In radiation-transfer calculations, we took into account the rotational band and the bands with the wavenumbers 7250, 5331, and 3755 cm^{-1} for water and the bands with the wavenumbers 667 and 3715 cm^{-1} for carbon dioxide.

The optical properties of the particles were described in [12], where the cloud of particles was considered as a polydisperse mixture with the gamma-distribution of the particle size, and approximate formulas were obtained for calculating the coefficients of attenuation and scattering as functions of the diffraction parameter:

$$x = \pi \bar d/\lambda$$

($\bar d$ is the mean particle diameter). Correspondingly, the expressions for the coefficients of attenuation and scattering of coal particles acquire the form

$$k_{\lambda p} = 2\pi N \Big(\frac{\bar d}{2}\Big)^2 \frac{\alpha + 2}{\alpha + 1}, \qquad \beta_{\lambda p} = \pi N \Big(\frac{\bar d}{2}\Big)^2 \frac{\alpha + 1}{\alpha + 2}\Big(2 - \frac{f_1 + f_2}{2}\Big),$$

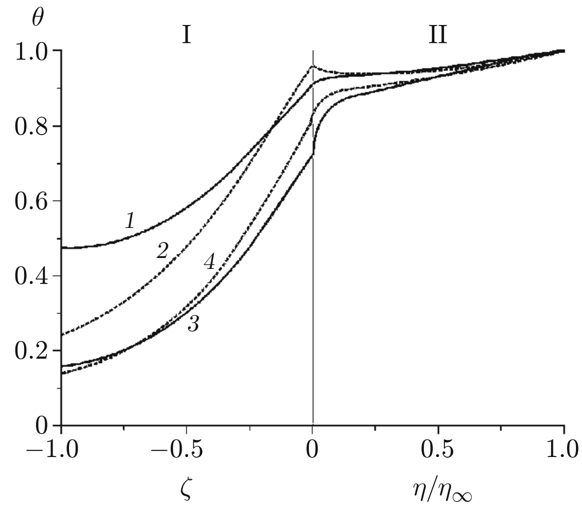

Fig. 1. Temperature distribution in the boundary layer and in the plate ($R_b = 0.0005$) in the presence of ablation (1 and 3) and in the absence of ablation (2 and 4) with 100 steps of integration with respect to time (1 and 2) and 20 steps of integration with respect to time (3 and 4): the regions corresponding to the plate and the boundary layer are indicated by I and II, respectively.

where N is the concentration of particles (number of particles in a unit volume), α is an empirical coefficient characterizing the particle size distribution,

$$f_i = 8[q_i - \ln(1 + q_i + q_i^2/2)]/q_i^2, \quad i = 1, 2, \qquad q_1 = (nn')^{-1/2}, \quad q_2 = 2/q_1,$$

n and n' are the real and imaginary parts of the complex refractive index $m = n - in'$, respectively.

The plate material was K-8 silica glass ($\lambda_c = 1.42$ W/mK; the refractive index equals 1.5). The data on absorption bands of this material were given in [5].

In the present work, we used the ablation model proposed in [4], which implies that evaporation is the governing destructive process in ablation. Evaporation is assumed to be essentially nonequilibrium, and the pressure of saturated vapors in the flow is significantly lower than the saturation pressure for all values of the surface temperature. Such a situation is typical of high-velocity flows. Numerous experiments on evaporation of various materials show that the mass flow of vapor as a function of the plate temperature can be approximately presented in the form of the Langmuir–Knudsen law for evaporation in essentially nonequilibrium conditions [13]:

$$V_w = \frac{a_1}{\sqrt{\theta_w}} \exp\Big(-\frac{a_2}{\theta_w}\Big).$$

Here a_1 and a_2 are coefficients independent of the plate temperature:

$$a_1 = \frac{aP_{\rm sat}(T_\infty)}{\rho_\infty u_\infty}\sqrt{\frac{M}{2\pi RT_\infty}}\,\exp\Big(\frac{q_L M}{RT_\infty}\Big), \qquad a_2 = \frac{q_L M}{RT_\infty},$$

a is the accommodation coefficient, $P_{\rm sat}$ is the pressure of saturated vapors, M is the molecular weight of the vapors, and R is the gas constant. The values of a_1 and a_2 vary within wide ranges for different materials.

The results presented in the paper were obtained for the external flow temperature $T_\infty = 1000$ K, temperature of the external source of radiation $T_s = 1500$ K, $a_1 = 10^{-3}$, $a_2 = 10^{-2}$, and $Q_L = 0.4$. The calculations were performed for the following values of the governing parameters: $\theta_{w0} = 0.2$, $\Pr = 0.7$, $\Pr_t = 0.9$, $\mathrm{Re} = 10^6$, $\mathrm{Sk} = 10^4$, $\mathrm{Sk}_w = 15$, and dimensionless time step $\Delta\,\mathrm{Fo} = 0.01$. The concentration of carbon dioxide was assumed to be $C_{CO_2} = 0$, and the concentration of water vapors was assumed to be $C_{H_2O} = 1$. The total pressure of the gas was 10^5 Pa, and coal particles with the mean diameter 10^{-4} m were considered as the solid particles in the flow. For coal particles, we assumed that $\alpha = 4$ and $m = 2.02 - 0.8i$; the value of the refractive index of the gas mixture was taken to be equal to 1.

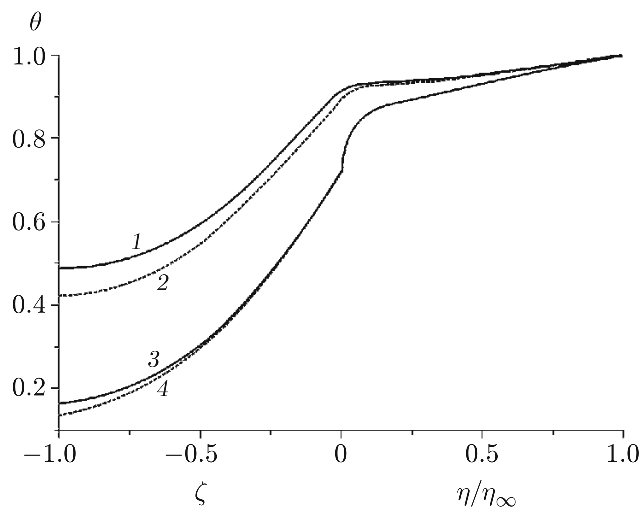

Fig. 2

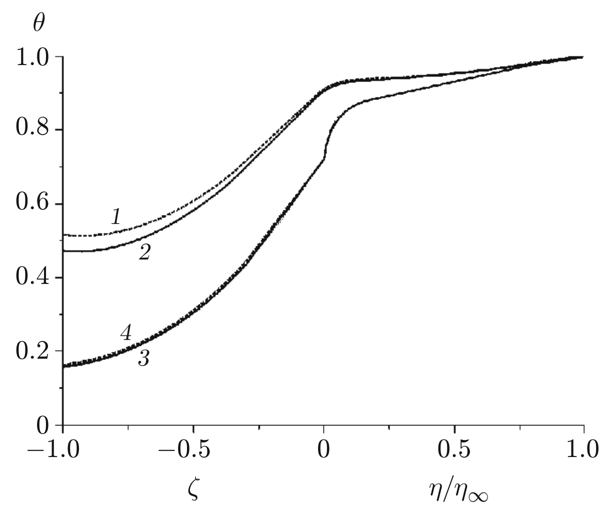

Fig. 3

Fig. 2. Temperature distribution in the system consisting of the boundary layer and the plate ($R_b = 0.0005$) versus the particle concentration in the boundary layer with 100 steps of integration with respect to time for $N = 0$ (1) and $8 \cdot 10^7$ m^{-3} (2) and with 20 steps of integration with respect to time for $N = 0$ (3) and $8 \cdot 10^7$ m^{-3} (4).

Fig. 3. Temperature distribution in the system consisting of the boundary layer and the plate versus the reflection coefficient of the lower surface of the plate with 100 steps of integration with respect to time for $R_b = 0.0005$ (1) and 0.01 (2) and with 20 steps of integration with respect to time for $R_b = 0.0005$ (3) and 0.01 (4).

For an ideal plate surface and above-given values of refractive indices, the Walsh–Duncle formula [14] predicts that the reflection coefficient on the interface between the boundary layer and the plate is $R_2 = 0.092$ on the side of the boundary layer. Using the relation for the balance of radiation energy on the interface [15], we obtain the reflection coefficient of the interface on the side of the plate: $R_1 = 0.6$. The reflection coefficient of the lower surface of the plate takes the values $R_b = 0.0005$ and 0.0100.

Let us consider the calculated temperature distributions in the last section of the system consisting of the boundary layer and the plate.

Figure 1 shows the temperature distribution calculated with and without allowance for ablation. It is seen that allowance for ablation leads to lower temperatures of the plate surface because of reduction of the heat flux from the boundary layer to the plate owing to heat losses for the phase transition. It is also seen that ablation is responsible for a smaller temperature gradient in the plate.

Figure 2 shows the effect of the particle concentration N in the flow on the temperature distribution. It is seen that the temperature in the plate decreases with increasing concentration. This is caused by attenuation of the radiation flux in the boundary layer owing to radiation absorption and scattering on particles. The temperature field in the boundary layer is more stable to changes in this parameter.

The influence of radiation reflection on the formation of the temperature field in the system consisting of the boundary layer and the plate is illustrated in Fig. 3. An increase in the reflection coefficient of the lower surface of the plate leads to more intense heating of the plate. This can be attributed to an increase in the number of radiation reflections inside the plate.

The analysis performed shows that the proposed model allows one to study the main features of heat and mass transfer in the boundary layer formed on a ablating semi-transparent flat plate in a flow of a high-temperature gas–particle medium with an external source of radiation.

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